Effects of Power-Law Distribution and Exponential with Uniform Pressures on the Vibration Behavior of Reinforced Cylindrical Shell Made of Functionally Graded Materials under Symmetric Boundary Conditions

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ABSTRACT

In this paper, the influence of the constituent volume fractions by changing the values of the power-law exponent with uniform pressure on the vibration frequencies of reinforced functionally graded cylindrical shells is studied. The FGM shell with ring is developed in accordance with the volume fraction law from two constituents namely stainless steel and nickel. These constituents are graded through the thickness direction, from one surface of the shell to the other, and are controlled by power-law volume fraction distribution. The reinforced FGM shell equations with ring and uniform pressure are established based on first order shear deformation theory. The governing equations of motion were employed using energy functional and by applying the Ritz method. The boundary conditions represented by end conditions of the FGM cylindrical shell are simply supported-simply supported, clamped-clamped and free-free. Effects of the different values of the power-law exponent, uniform pressure, reinforced ring and different symmetric boundary conditions on natural frequencies characteristics were also studied. To check the validity of the present study, the results obtained were compared with those available in the literature.

1. Introduction

Shells as structural elements are found in engineering and industrial fields since they improve favorable conditions for dynamic behavior, strength and stability. The study of the vibration of shells is an important issue for successful applications of these structures.

A special kind of shells is a cylindrical shell. Many applications of cylindrical shells are found in engineering from large aerospace, naval construction, civil and mechanical structures to small electrical components [1]. They are used as structures in aircrafts, ships, rockets, submarines, missile bodies, pressure vessels, and buildings, etc. For increase stiffness cylindrical shells and to avoid premature failure, stiffeners are used [2-7]. Vibration behavior of cylindrical shells is an important area of research in structural dynamics and the characteristics of cylindrical shells have been studied by many researchers. It was first introduced by Love [8]. Leissa [9] presented various theories for the vibration of cylindrical shells. Analysis of natural frequencies and mode shapes of cylindrical shells was reported by Blevins [10]. Soedel [11] and Chung [12] worked on the vibration of circular cylindrical shells. Reddy [13] discussed the thickness changes of cylindrical shells and plates under vibration. Forsberg [14] studied the effect of boundary conditions on natural frequencies.

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Functionally graded materials (FGMs) are fabricated by combining disparate materials which are graded in the thickness direction with variations in constituent volume fractions. These materials consist of a mixture of ceramic and metal, or a combination of different materials. The main advantage of using FGMs is that they can be used in environments with high temperature. FGM shells structures are generally used as structural components in missiles engines, resistant coatings in space plans, atomic reactors, spacecraft thermal shields, intelligent electrical components, submarines, turbine components, and so on [15]. Study on the vibration of cylindrical shells made-up of FGM is important in engineering applications. Since 1999, there are several reviews on vibration of FGM cylindrical shells without pressure. The first works on vibration analysis of FGM cylindrical shell was reported by Loy et al. [16]. They analyzed the natural frequency by constituent different volume fractions with simply supported boundary condition. Patel et al. [17] used finite element method for vibration analysis of FGM in cylindrical shells. Zhi and Hua [18] studied the natural frequencies of FGM cylindrical shell with cavities and effects of radius to span ratio. In FGMs, material distribution is controlled by the exponential volume fraction and this distribution leads to continuous change in the combination of the shell and results in gradient mechanical and thermal properties. Arshad et al. [19] analyzed the frequency characteristic of FGM cylindrical shell by assuming mathematical forms of the volume fraction law. Shah et al. [20] applied modified volume fraction law for the fabrication of FGM cylindrical shell with simply supported boundary condition. Reported works on vibration of reinforced FGM cylindrical shell for changing the values of the power-law exponents with uniform pressure could not be found in the literature.

The aim of this paper is to study the influence of constituent volume fractions with uniform pressure by changing the values of the power-law exponents on the vibration behavior of reinforced FGM cylindrical shells. The analysis is carried out based on the first order shear deformation theory. The governing equations of motion are derived using Ritz method with energy functional. The boundary conditions of supported FGM cylindrical shell considered are the combination of simply supported-simply supported (SS-SS), clamped-clamped (C-C), and free-free (F-F). The influence of different values of the power-law exponent with uniform pressure and one ring, and the effect of the considered different boundary conditions on the natural frequencies are discussed. The validity and accuracy of the present method is checked by comparing the present results with those in the literature.

2. Materials and methods

2.1. Modeling of Functionally Graded Materials

Functionally graded materials (FGMs) are made up of variation of composition and different materials. The volume fraction distribution of each phase of material varies with a specific gradient in the thickness direction, thus the properties of functionally graded materials change along this direction. There are two possible structures of FGMs as shown in Fig. 1. For the first type, the volume fraction changes stepwise as shown in Fig. 1(a), while the second type, the variation is continuous as shown in Fig. 1(b).

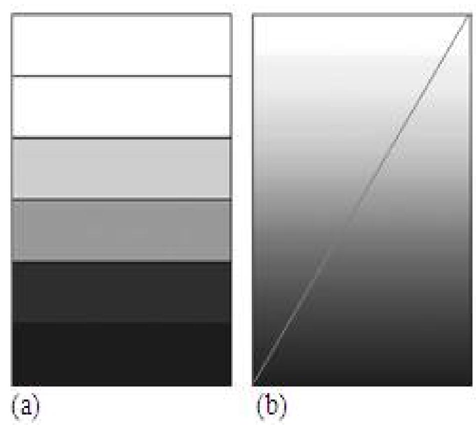

Fig. 1. Structure of the functionally graded material.

The effective material properties, Q_{fgm} of an FGM, depend on its properties and the volume fractions of the constituent materials, and it is defined as [21]:

$$Q_{fgm}(T,z)=\sum_{j=1}^{k}\overline{Q}_j(T)V_{fj}(z) \qquad (1)$$

where $\overline{Q}_j(T)$ is the material property and $V_{fj}(z)$ is the volume fraction for the constituent material *j*.

For FGM cylindrical shell made of two different materials, the volume fractions, $V_{f1}(z)$ and $V_{f2}(z)$, are expressed as [21]:

$$V_{f2}(z)=\left(\frac{z+h/2}{h}\right)^N \quad V_{f1}(z)=1-\left(\frac{z+h/2}{h}\right)^N \tag{2}$$

$$V_{f1}(z)+V_{f2}(z)=1 \tag{3}$$

where the power-law exponent *N* is a real value, $0\le N\le\infty$, and *z* denotes the radial distance measured from mid-surface of the FGM shell ($-h/2\le z\le h/2$).

Structures using FGMs are generally used in high temperature environments and their material properties are temperature dependent. Material properties $\overline{Q}_j(T)$ can be described as a function of temperature:

$$\overline{Q}_j(T)=Q_{0,j}(Q_{-1,j}T^{-1}+1+Q_{1,j}T+Q_{2,j}T^2+Q_{3,j}T^3) \tag{4}$$

where $Q_{0,j}, Q_{-1,j}, Q_{1,j}, Q_{2,j}$ and $Q_{3,j}$ are the temperature coefficients of the constituent material *j*.

The materials of the FGM cylindrical shell considered in this study are composed of stainless steel and nickel, with the Young modulus, *E*, Poisson ratio, *ν*, and the mass density, ρ, is defined as:

$$E_{fgm}(T,z)=(E_2(T)-E_1(T))\left(\frac{z+\frac{h}{2}}{h}\right)^N+E_1(T) \tag{5}$$

$$\nu_{fgm}(T,z)=(\nu_2(T)-\nu_1(T))\left(\frac{z+\frac{h}{2}}{h}\right)^N+\nu_1(T) \tag{6}$$

$$\rho_{fgm}(T,z)=(\rho_2(T)-\rho_1(T))\left(\frac{z+\frac{h}{2}}{h}\right)^N+\rho_1(T) \tag{7}$$

2.2. First Order Shear Deformation Theory

Consider a reinforced cylindrical shell made of functionally graded material (FGM) with one ring subjected to uniform pressure with the thickness *h*, radius of the shell *R*, length *L*, position of the ring support *b*, uniform pressure *P*, mass density ρ, modulus of elasticity *E*, and Poisson ratio *ν*, as displayed in Fig. 2. The deformation of functionally graded material cylindrical shell is defined with reference to the coordinate system (*x*, *θ*, *z*) in which *x* and *θ* are axial and circumferential directions of the functionally graded cylindrical shell and *z* is in the radial direction to mid-surface. The corresponding displacements on the mid-surface of FGM cylindrical shell are defined by *u*, *v*, and *w*.

The displacement fields based on first order shear deformation theory for an arbitrary point

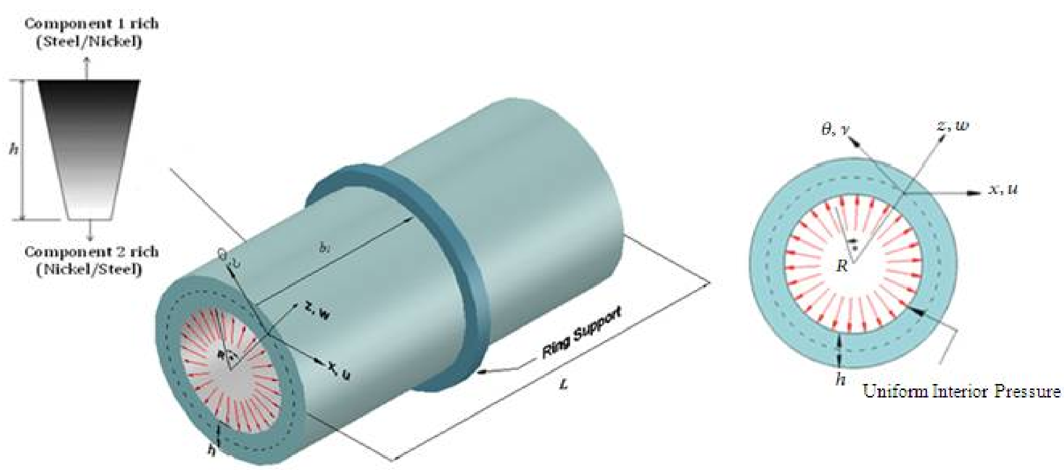

Fig. 2. Geometry of the reinforced FGM cylindrical shell with one ring subjected to uniform pressures.

in the cylindrical coordinate system under the Kirchhoff hypothesis are expressed as follows:

$$
\begin{aligned}
u(x,\theta,z) &= u_0(x,\theta) + z\psi_x(x,\theta)\\
v(x,\theta,z) &= v_0(x,\theta) + z\psi_\theta(x,\theta)\\
w(x,\theta,z) &= w_0(x,\theta)
\end{aligned}
\tag{8}
$$

where $u(x,\theta,z)$, $v(x,\theta,z)$, and $w(x,\theta,z)$ are the components of displacement in x, θ and z direction respectively, $u_0(x,\theta)$, $v_0(x,\theta)$ and $w_0(x,\theta)$ are the displacements of the mid-surface of the FGM cylindrical shell, and $\psi_x(x,\theta)$, $\psi_\theta(x,\theta)$ are the rotations of the normal to the mid-surface of the FGM cylindrical shell about the *x* and *θ* axes, respectively.

The strain-displacement relationships for FGM cylindrical shell are expressed by:

$$\bar{\varepsilon}_{11} = \frac{1}{A_1}\frac{\partial u(x,\theta,z)}{\partial x} + \frac{1}{A_1A_2}\frac{\partial A_1}{\partial \theta} v(x,\theta,z) + \frac{w(x,\theta,z)}{R_1} \tag{9}$$

$$\bar{\varepsilon}_{22} = \frac{1}{A_2}\frac{\partial v(x,\theta,z)}{\partial \theta} + \frac{1}{A_1A_2}\frac{\partial A_2}{\partial x} u(x,\theta,z) + \frac{w(x,\theta,z)}{R_2} \tag{10}$$

$$\bar{\varepsilon}_{12} = \frac{A_2}{A_1}\frac{\partial}{\partial x}\left(\frac{v(x,\theta,z)}{A_2}\right) + \frac{A_1}{A_2}\frac{\partial}{\partial \theta}\left(\frac{u(x,\theta,z)}{A_1}\right) \tag{11}$$

$$\bar{\varepsilon}_{13} = A_1\frac{\partial}{\partial z}\left(\frac{u(x,\theta,z)}{A_1}\right) + \frac{1}{A_1}\frac{\partial w(x,\theta,z)}{\partial x} \tag{12}$$

$$\bar{\varepsilon}_{23} = A_2\frac{\partial}{\partial z}\left(\frac{v(x,\theta,z)}{A_2}\right) + \frac{1}{A_2}\frac{\partial w(x,\theta,z)}{\partial \theta} \tag{13}$$

$$\bar{\varepsilon}_{33} = 0 \tag{14}$$

where A_1 and A_2 are the parameters of Lame for FGM cylindrical shell and defined by the following Soedel formulas [22].

$$A_1 = \frac{\partial r}{\partial x},\ A_2 = \frac{\partial r}{\partial \theta} \tag{15}$$

$$\bar{\varepsilon}_{11} = \frac{\partial u_0(x,\theta)}{\partial x} + z\frac{\partial \psi_x(x,\theta)}{\partial x} \tag{16}$$

$$\bar{\varepsilon}_{22} = \frac{\partial v_0(x,\theta)}{R\partial \theta} + z\frac{\partial \psi_\theta(x,\theta)}{R\partial \theta} + \frac{w_0(x,\theta)}{R} \tag{17}$$

$$\bar{\varepsilon}_{12} = \frac{\partial v_0(x,\theta)}{\partial x} + \frac{\partial u_0(x,\theta)}{R\partial \theta} + z\left(\frac{\partial \psi_x(x,\theta)}{R\partial \theta} + \frac{\partial \psi_\theta(x,\theta)}{\partial x}\right) \tag{18}$$

$$\bar{\varepsilon}_{13} = \psi_x(x,\theta) + \frac{\partial w_0(x,\theta)}{\partial x} \tag{19}$$

$$\bar{\varepsilon}_{23} = \psi_\theta(x,\theta) + \frac{\partial w_0(x,\theta)}{R\partial \theta} \tag{20}$$

The stress-strain relation for a FGM cylindrical shell with plane-stress condition is expressed by:

$$\{\bar{\sigma}\} = \left[\bar{Q}\right]\{\bar{\varepsilon}\} \tag{21}$$

where $\{\bar{\sigma}\}$, $\{\bar{\varepsilon}\}$ are the corresponding stress and strain vectors, respectively, and $\left[\bar{Q}\right]$ is the reduced stiffness matrix expressed as:

$$\{\bar{\sigma}\}^T = \{\bar{\sigma}_{11}\ \bar{\sigma}_{22}\ \bar{\sigma}_{12}\ \bar{\sigma}_{13}\ \bar{\sigma}_{23}\} \tag{22}$$

Substituting Eq.(8) into strain-displacement relationships (Eqs.(9)-(13)), and applying the cylindrical coordinate system, thus

$$\{\bar{\varepsilon}\}^T = \{\bar{\varepsilon}_{11}\ \bar{\varepsilon}_{22}\ \bar{\varepsilon}_{12}\ \bar{\varepsilon}_{13}\ \bar{\varepsilon}_{23}\} \tag{23}$$

$$\left[\bar{Q}\right] = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 & 0 & 0\\ \bar{Q}_{21} & \bar{Q}_{22} & 0 & 0 & 0\\ 0 & 0 & \bar{Q}_{66} & 0 & 0\\ 0 & 0 & 0 & \bar{Q}_{55} & 0\\ 0 & 0 & 0 & 0 & \bar{Q}_{44} \end{bmatrix} \tag{24}$$

Then equation (21) can be expressed as:

$$\begin{Bmatrix} \bar{\sigma}_{11}\\ \bar{\sigma}_{22}\\ \bar{\sigma}_{12}\\ \bar{\sigma}_{13}\\ \bar{\sigma}_{23} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 & 0 & 0\\ \bar{Q}_{21} & \bar{Q}_{22} & 0 & 0 & 0\\ 0 & 0 & \bar{Q}_{66} & 0 & 0\\ 0 & 0 & 0 & \bar{Q}_{55} & 0\\ 0 & 0 & 0 & 0 & \bar{Q}_{44} \end{bmatrix} \begin{Bmatrix} \bar{\varepsilon}_{11}\\ \bar{\varepsilon}_{22}\\ \bar{\varepsilon}_{12}\\ \bar{\varepsilon}_{13}\\ \bar{\varepsilon}_{23} \end{Bmatrix}$$

(25)

For FGM cylindrical shells, the stiffness $\bar{Q}_{ij}$ is defined as:

$$\bar{Q}_{11} = \frac{E(z)}{1-\nu^2(z)},\ \bar{Q}_{12} = \frac{\nu(z)E(z)}{A(1-\nu^2(z))} \tag{26}$$

$$\bar{Q}_{21} = \frac{\nu(z)E(z)}{1-\nu^2(z)},\ \bar{Q}_{22} = \frac{E(z)}{A(1-\nu^2(z))} \tag{27}$$

$$\bar{Q}_{66} = \frac{E(z)}{2A(1-\nu(z))},\ \bar{Q}_{44} = K\frac{E(z)}{2(1-\nu(z))} \tag{28}$$

$$\bar{Q}_{55} = K\frac{E(z)}{2(1-\nu(z))},\quad A = 1+\frac{z}{R} \tag{29}$$

where K is the shear correction coefficient and is taken as K = 5/6 [23].

The stress and moment resultants are defined respectively as:

$$\{N_x, N_\theta, N_{x\theta}\ H_x\ H_\theta\} = \int_{-h/2}^{h/2}\{\bar{\sigma}_{11}\ \bar{\sigma}_{22}\ \bar{\sigma}_{12}\ \bar{\sigma}_{13}\ \bar{\sigma}_{23}\}dz \tag{30}$$

$$\{M_x, M_\theta, M_{x\theta}\} = \int_{-h/2}^{h/2} \{\bar{\sigma}_{11}\ \bar{\sigma}_{22}\ \bar{\sigma}_{12}\} z\, dz \quad (31)$$

Applying Eqs.(16)-(20) into Eq.(25) and then substituting into Eqs.(30)-(31) the stress and moment resultants are combined as:

$$\{N\} = [I]\{\bar{\varepsilon}\} \quad (32)$$

where $\{N\}$ and $\{\bar{\varepsilon}\}$ are expressed as:

$$\{N\}^T = \{N_x, N_\theta, N_{x\theta}, M_x, M_\theta, M_{x\theta}\ H_x\ H_\theta\} \quad (33)$$

$$\{\bar{\varepsilon}\}^T = \{\bar{\varepsilon}_{11}\ \bar{\varepsilon}_{22}\ \bar{\varepsilon}_{12}\ \bar{\varepsilon}_{11}\ \bar{\varepsilon}_{22}\ \bar{\varepsilon}_{12}\ \bar{\varepsilon}_{13}\ \bar{\varepsilon}_{23}\} \quad (34)$$

$[I]$ is the matrix of stiffness and can be written as:

$$[I] = \begin{bmatrix} X_{11} & X_{12} & X_{16} & Y_{11} & Y_{12} & Y_{16} & 0 & 0 \\ X_{12} & X_{22} & X_{26} & Y_{12} & Y_{22} & Y_{26} & 0 & 0 \\ X_{16} & X_{26} & X_{66} & Y_{16} & Y_{26} & Y_{66} & 0 & 0 \\ Y_{11} & Y_{12} & Y_{16} & Z_{11} & Z_{12} & Z_{16} & 0 & 0 \\ Y_{12} & Y_{22} & Y_{26} & Z_{12} & Z_{22} & Z_{26} & 0 & 0 \\ Y_{16} & Y_{26} & Y_{66} & Z_{16} & Z_{26} & Z_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & kV_{44} & kV_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & kV_{45} & kV_{55} \end{bmatrix} \quad (35)$$

where X_{ij}, Y_{ij}, Z_{ij} are the extensional, coupling, and bending stiffness matrices, and V_{ij} is thickness shear stiffness matrices and is defined as:

$$(X_{ij}, Y_{ij}, Z_{ij}) = \int_{-h/2}^{h/2} Q_{ij}(1, Z, Z^2)dz\ ,\ V_{ij} = K\int_{-h/2}^{h/2} Q_{ij}dz \quad (36)$$

2.3. Energy Equations

The expressions for the strain energy, the potential energy of uniform pressure and the kinetic energy depend on the theory chosen to describe the vibration of FGM cylindrical shell.

2.3.1. Strain energy

Based on first order shear deformation theory the strain energy U is expressed as:

$$U = \frac{1}{2}\int_0^L \int_0^{2\pi} \{\bar{\varepsilon}\}^T [I]\{\bar{\varepsilon}\} R d\theta\, dx \quad (37)$$

2.3.2. Kinetic energy

Based on first order shear deformation theory the kinetic energy for vibration of FGM cylindrical shell during is given by:

$$T = \frac{1}{2}\int_0^L \int_0^{2\pi} \rho_T \{ (\frac{\partial u_0(x,\theta)}{\partial t})^2 + (\frac{\partial v_0(x,\theta)}{\partial t})^2 + (\frac{\partial \omega_0(x,\theta)}{\partial t})^2 + (\frac{\partial \psi_x(x,\theta)}{\partial t})^2 + (\frac{\partial \psi_\theta(x,\theta)}{\partial t})^2 \} R d\theta dx \quad (36)$$

where ρ_T is the density of unit length, and is defined as:

$$\rho_T = \int_{-h/2}^{h/2} \rho\, dz \quad (39)$$

2.3.3. Uniform pressure distribution energy

The potential energy of the uniform pressure P for FGM cylindrical shell based on first order shear deformation theory is:

$$V = \int_0^L \int_0^{2\pi} \frac{P}{2} \left\{ \left[\frac{\partial^2 w_0(x,\theta)}{\partial \theta^2} + w_0(x,\theta) \right] w_0(x,\theta) \right\} d\theta dx \quad (40)$$

Therefore, the energy functional for vibration of FGM cylindrical shell with reinforced and uniform pressure can be written as:

$$F = U - T + V \quad (41)$$

2.4. The Displacement Field

The displacement field for vibration of FGM cylindrical shell with a number of reinforced and uniform pressures can be expressed as:

$$\begin{aligned} u_0(x,\theta) &= \bar{E}_1 \frac{\partial \Omega(x)}{\partial x} \cos(n\theta)\cos(\omega t) \\ v_0(x,\theta) &= \bar{E}_2 \Omega(x) \sin(n\theta)\cos(\omega t) \\ w_0(x,\theta) &= \bar{E}_3 \Omega(x) \prod_{i=1}^{H} (x - b_i)^{\mu_i} \cos(n\theta) \cos(\omega t) \\ \psi_x(x,\theta) &= \bar{E}_4 \frac{\partial \Omega(x)}{\partial x} \cos(n\theta)\cos(\omega t) \\ \psi_\theta(x,\theta) &= \bar{E}_5 \Omega(x) \sin(n\theta)\cos(\omega t) \end{aligned} \quad (42)$$

where $\bar{E}_1, \bar{E}_2, \bar{E}_3$, $\bar{E}_4$ and $\bar{E}_5$ are the constants denoting the vibrational amplitude, $\Omega(x)$ is the axial function that satisfies the boundary conditions, b_i is the position of reinforced ring, H is the number of reinforced, μ_i is a parameter having a value of 1 when there is one ring, n is the circumferential waves number, and ω is the natural frequency.

The axial function $\Omega(x)$ selected as the beam function is given by Moon and Shaw [24]:

$$(43)\quad \Omega(x) = \Psi_1 \cosh(\frac{\Phi_m x}{L}) + \Psi_2 \cos(\frac{\Phi_m x}{L}) - \mu_m(\Psi_3 \sinh(\frac{\Phi_m x}{L}) + \Psi_4 \sin(\frac{\Phi_m x}{L}))$$

where the values of $\Psi_i (i = 1,...,4)$, Φ_m and μ_m for FGM cylindrical shell with one reinforced and uniform pressure for different symmetric boundary conditions are given in Table 1. In this Table, m represents the axial wave number. The symmetric boundary conditions for simply supported, free, and clamped that satisfy x = 0 and x = L can be expressed as:

Simply supported boundary condition

$$\Omega(0) = \frac{\partial^2 \Omega(L)}{\partial x^2} = 0 \tag{44}$$

Free boundary condition

$$\frac{\partial^2 \Omega(0)}{\partial x^2} = \frac{\partial^3 \Omega(L)}{\partial x^3} = 0 \tag{45}$$

Clamped boundary condition

$$\Omega(0) = \frac{\partial \Omega(L)}{\partial x} = 0 \tag{46}$$

2.5. The Ritz Method

To determine the natural frequency of FGM cylindrical shell with reinforced and pressure, the Ritz technique is used. The energy functional, F, is defined by the Lagrangian function as:

$$F = U_{max} - T_{max} + V_{max} \tag{47}$$

Substituting Eq. (42) into Eqs. (37), (38), and (40) and applying the Ritz technique with minimizing the energy functional F, we have:

$$\left.\begin{aligned}
\frac{\partial(U_{max} - T_{max} + V_{max})}{\partial \bar{E}_1} = 0 \\
\frac{\partial(U_{max} - T_{max} + V_{max})}{\partial \bar{E}_2} = 0 \\
\frac{\partial(U_{max} - T_{max} + V_{max})}{\partial \bar{E}_3} = 0 \\
\frac{\partial(U_{max} - T_{max} + V_{max})}{\partial \bar{E}_4} = 0 \\
\frac{\partial(U_{max} - T_{max} + V_{max})}{\partial \bar{E}_5} = 0
\end{aligned}\right\} \tag{48}$$

There are five equations of motion in Eq. (48) characterizing the vibration characteristic of FGM cylindrical shell with uniform pressure and reinforced pressure. Therefore, the governing eigenvalue equation can be written in a matrix form as:

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} \end{bmatrix} \begin{Bmatrix} \bar{E}_1 \\ \bar{E}_2 \\ \bar{E}_3 \\ \bar{E}_4 \\ \bar{E}_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{49}$$

The solution is obtained by setting the determinant of matrix C as equal to zero, i.e.:

Table 1. The values of Ψ_i, Φ_m and μ_m for symmetric boundary conditions

Symmetric Boundary Conditions	$\Psi_i (i = 1,...,4)$	Φ_m	μ_m
Simply Support-Simply Support (SS-SS)	$\Psi_1 = 0$, $\Psi_2 = 0$ $\Psi_3 = 0$, $\Psi_4 = -1$	$m\pi$	1
Clamped-Clamped (C-C)	$\Psi_1 = 1$, $\Psi_2 = -1$ $\Psi_3 = 1$, $\Psi_4 = -1$	$(2m+1)\pi / 2$	$\frac{\cosh \Phi_m - \cos \Phi_m}{\sinh \Phi_m - \sin \Phi_m}$
Free-Free (F-F)	$\Psi_1 = 1$, $\Psi_2 = 1$ $\Psi_3 = 1$, $\Psi_4 = 1$	$(2m+1)\pi / 2$	$\frac{\cosh \Phi_m - \cos \Phi_m}{\sinh \Phi_m - \sin \Phi_m}$

$$\left|C_{ij}\right| = 0 \qquad (i, j = 1,2,3,4,5) \tag{50}$$

The solution for equation (50) is obtained and the characteristic of the FGM cylindrical shell with uniform pressure and reinforced is expressed in the power of ω as

$$\delta_{\circ}\omega^{10} + \delta_1\omega^8 + \delta_2\omega^6 + \delta_3\omega^4 + \delta_4\omega^2 + \delta_5 = 0 \tag{51}$$

The solution of equation (51) consists of ten roots, and the five positive roots are the natural frequencies. The smallest positive root is the frequency of interest in the present work.

The functional graded material considered in this study is composed of stainless steel and nickel and the properties of the constituent materials are given in Table 2.

2.6. Comparison Study

In order to validate the accuracy of the present analysis, the results for FGM cylindrical shell without uniform pressure distribution and reinforced pressure are compared with the results available in the literature. Table 3 shows the variation of natural frequency with the circumferential wave numbers for FGM cylindrical shell without uniform pressure and reinforced pressure with two different h/R ratios. The comparisons presented in Table 3 show good agreeable results with the published works.

3. Results and Discussion

In this study, a reinforced FGM cylindrical shell according to different values of the power-law exponent with pressure is presented. The influence of constituent volume fractions is studied by varying the values of the power-law exponents (N) used for stainless steel and nickel.

3.1. Variation of the volume fraction of the FGM

Variations of the volume fractions V_f of stainless steel and nickel for constituent materials placed at the inner and the outer shell surfaces in the FGM shell layers are shown in Figs. 3 and 4.

In Fig. 3, the volume fraction of the constituent material nickel V_{fN} decreased from its maximum value 1 at thickness variable z/h = -0.5 to its minimum value 0 at thickness variable z/h = +0.5. In Fig. 4, the volume fraction of the constituent material stainless steel V_{fss} increased from its minimum value 0 at thickness variable z/h = -0.5 to its maximum value 1 at thickness variable z/h = +0.5. In Fig. 3, for z/h < 0 and N < 1, the rate of decrease of V_{fN} is rapid while for z/h > 0 and N < 1, it decreases slowly. For z/h < 0 and N > 1, the rate of decrease of V_{fN} is slow while for z/h > 0 and N > 1, it decreases rapidly.
In Fig. 4, for z/h < 0 and N < 1, the rate of increases of V_{fss} is rapid while for z/h > 0 and N < 1, it increases slowly. For z/h < 0 and N > 1, the rate of increases of V_{fss} is slow while for z/h > 0 and N > 1, it increases rapidly. From these figures, it is observed that the variations of constituent material for FGMs are influenced by volume fraction laws.

Table 2. Mechanical properties of constituent materials for FGM cylindrical shell [16]

Coefficients of temperature	Stainless Steel			Nickel		
	$E(Nm^{-2})$	v	$\rho(kgm^{-3})$	$E(Nm^{-2})$	v	$\rho(kgm^{-3})$
Q_0	201.04×10^9	0.3262	8166	223.95×10^9	0.3100	8900
Q_{-1}	0	0	0	0	0	0
Q_1	3.079×10^{-4}	-2.002×10^{-4}	0	-2.794×10^{-4}	0	0
Q_2	-6.534×10^{-7}	3.797×10^{-7}	0	-3.998×10^{-9}	0	0
Q_3	0	0	0	0	0	0
Q	2.07788×10^{11}	0.317756	8166	2.05098×10^{11}	0.3100	8900

3.2. Variation of natural frequencies with different values of the power-law exponent

Tables 4-6 show the variation of the natural frequencies with the circumferential wave numbers for reinforced FGM cylindrical shell with different power-law exponent under pressures. The natural frequencies have been considered for the values of the different power-law exponent N = 0.5, 2, 5, 15, 30, respectively. In these tables, the analyses are conducted by assuming the pressure as equal to 800 kPa and the reinforced position is placed along the length, i.e. at b = 0.5 L.

It is seen from these Tables that different values of power-law exponent pressures affect the natural frequency of reinforced FGM cylindrical shell. As material properties of the reinforced FGM cylindrical shells are graded in the thickness direction according to a volume fraction power-law distribution with pressure, therefore, due to their configuration, the natural frequencies decreased with the increase in the values of the power-law exponent for different boundary conditions.

The decrease in the natural frequencies in all boundary conditions from N = 0.5 to N = 30 is about 3.15% at n = 1 and about 3.09% at n = 10. Thus, the ordering of constituent materials in FGMs will determine the increment and decrements in the natural frequency with power-law exponents. The results show that the shell frequency varies with the FGM distribution determined by the values of the power-law exponent with pressure. The obtained results also show that the natural frequency characteristics of a reinforced FGM cylindrical shell for different values of the power-law exponent with pressure are different for different boundary conditions.

In Fig. 5, variation of natural frequencies of reinforced FGM cylindrical shell is plotted against the values of the power-law exponent with uniform pressure for SS–SS, C–C and F–F boundary conditions for circumferential wave number n = 5. It is observed that the natural frequencies decrease under the power-law exponent with uniform pressure for all the boundary conditions. Moreover, natural frequency all the boundary condition behave alike and natural frequency responses of the reinforced FGM cylindrical shells under uniform pressure with various boundary conditions change for different values of circumferential wave number.

For circumferential wave number n = 5, the natural frequencies of simply supported-simply supported boundary condition for reinforced FGM cylindrical shell under uniform pressures is higher and free-free boundary condition for reinforced FGM cylindrical shell under uniform pressure is lower than any other boundary conditions.

It must be noted that in Fig. 5, natural frequency response of clamped-clamped boundary condition is very close to that of simply supported-simply supported boundary condition for a reinforced FGM cylindrical shell with pressures.

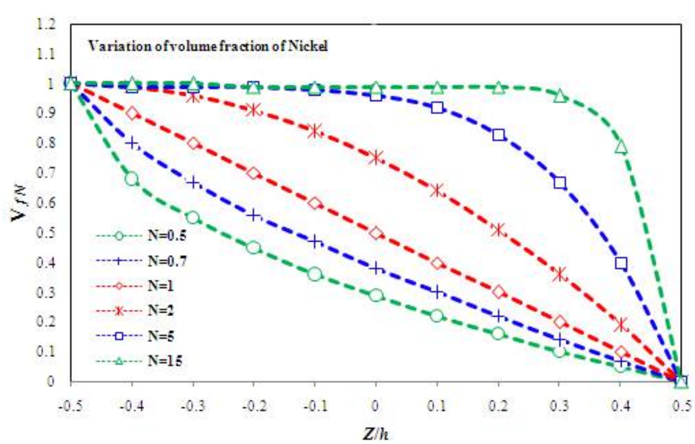

Fig. 3. Variation of volume fraction of Nickel V_{fN} with thickness variable Z/h for reinforced FGM cylindrical shell with uniform pressure.

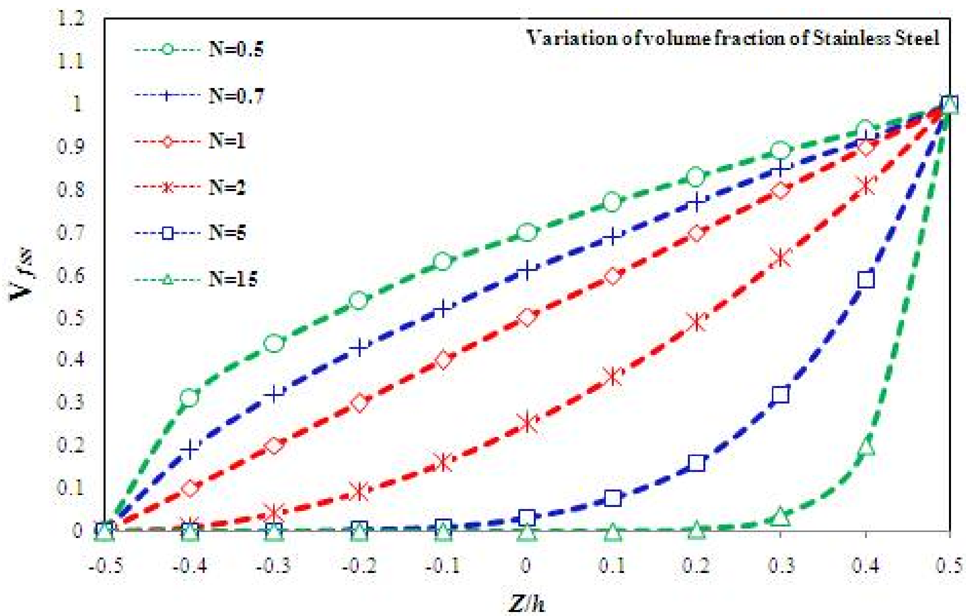

Fig. 4. Variation of volume fraction of Stainless Steel V_{fss} with thickness variable Z/h for reinforced FGM cylindrical shell with uniform pressure.

Table 4. Variation of the natural frequency with different values of the power-law exponent with uniform pressure for reinforced FGM cylindrical shell under SS-SS boundary conditions (h/R = 0.002, L/R = 20)

n	*m*	*P* = 800 kPa, *a*/*L* = 0.5				
		N = 0.5	*N* = 2	*N* = 5	*N* = 15	*N* = 30
1	1	492.193	484.134	480.225	477.821	477.129
2	1	834.684	821.028	814.402	810.325	809.151
3	1	838.301	824.603	817.953	813.860	812.681
4	1	843.351	829.594	822.911	818.795	817.608
5	1	849.819	835.986	829.260	825.115	823.919
6	1	857.687	843.762	836.984	832.803	831.597
7	1	866.936	852.901	846.062	841.840	840.620
8	1	877.541	863.380	856.472	852.201	850.966
9	1	889.478	875.175	868.187	863.861	862.610
10	1	902.721	888.258	881.182	876.796	875.526

Table 5. Variation of the natural frequency with different values of the power-law exponent with uniform pressure for reinforced FGM cylindrical shell under C-C boundary conditions (h/R = 0.002, L/R = 20)

n	*m*	*P* = 800 kPa, *a*/*L* = 0.5				
		N = 0.5	*N* = 2	*N* = 5	*N* = 15	*N* = 30
1	1	504.649	496.386	492.378	489.914	489.204
2	1	833.806	820.164	813.545	809.472	808.299
3	1	837.677	823.989	817.344	813.254	812.075
4	1	842.779	829.032	822.353	818.240	817.054
5	1	849.271	835.447	828.726	824.583	823.388
6	1	857.153	843.237	836.463	832.285	831.079
7	1	866.412	852.386	845.552	841.331	840.113
8	1	877.026	862.874	855.969	851.701	850.467
9	1	888.971	874.676	867.692	863.369	862.118
10	1	902.220	887.766	880.693	876.310	875.041

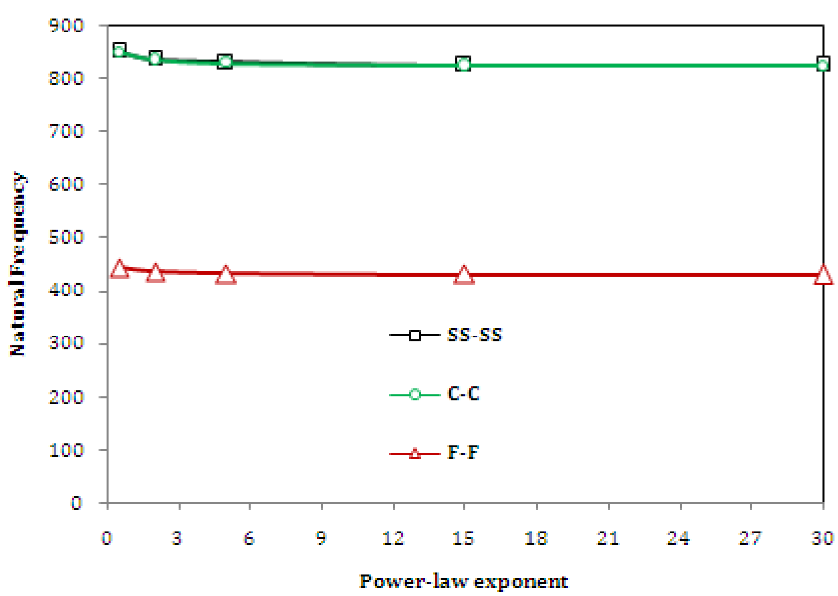

Fig. 5. Variation of natural frequency of reinforced FGM cylindrical shell with power-law exponent with pressure for different boundary conditions
(m = 1, h/R = 0.002, L/R = 20, P = 800 kPa, a/L = 0.5)

4. Conclusions

This study presents the effect of constituent volume fractions by changing the values of the power-law exponent with uniform pressure on the vibration of reinforced FGM cylindrical shells for different boundary conditions. The FGM is made up of a distribution of stainless steel and nickel, and the material properties are graded along the thickness direction, according to a volume fraction power-law exponent. The first order shear deformation theory is employed and the governing equations of motion were derived, using energy functional applied in Ritz method.

Natural frequencies of reinforced FGM cylindrical shell change with volume fraction power-law exponent with uniform pressure for the symmetric boundary conditions. The symmetric boundary conditions are simply supported-simply supported, clamped-clamped, and free-free.

It is observed that material distribution is controlled by the volume fraction power-law with uniform pressures, and affects the natural frequencies of a reinforced FGM cylindrical shell. Natural frequencies of the reinforced FGM cylindrical shell with different values of the power-law exponent with uniform pressure for different boundary conditions are affected by the variation of circumferential wave number.

This study also shows that the natural frequency response decreased with the increase in the values of the power-law exponent with uniform pressure for different boundary conditions. Thus, the constituent volume fraction power-law exponent with uniform pressures affects the natural frequencies of reinforced FGM cylindrical shell.

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